UNIT - I

PROPERTIES OF MATTER

ELASTICITY

When an external force is applied on a body, which is not free to move, there will be a relative displacement of the particles. Due to the property of elasticity, the particles tend to regain their original position. The external forces may produce change in length, volume and shape of the body. This external force which produces these changes in the body is called deforming force. A body which experiences such a force is called deformed body. When the deforming force is removed, the body regains its original state due to the force developed within the body. This force is called restoring force. The property of a material to regain its original state when the deforming force is removed is called elasticity. The bodies which possess this property are called elastic bodies. Bodies which do not exhibit the property of elasticity are called plastic. The study of mechanical properties helps us to select the material for specific purposes. For example, springs are made of steel because steel is highly elastic.

Stress and strain

In a deformed body, restoring force is set up within the body which tends to bring the body back to the normal position. The magnitude of these restoring force depends upon the deformation caused. This restoring force per unit area of a deformed body is known as stress.

$$\text{Stress} = \frac{\text{Restoring force}}{\text{Area}} \ \text{Nm}^{-2}$$

Its dimensional formula is $ML^{-1}T^{-2}$

Due to the application of deforming force, length, volume or shape of a body changes. Or in other words, the body is said to be strained. Thus, strain produced in a body is defined as the ratio of change in dimension of a body to the original dimension.

$$\text{Strain} = \frac{\text{Change in dimension}}{\text{Original dimension}}$$

Strain is the ratio of two similar quantities. Therefore it has no unit.

Elastic limit

If an elastic material is stretched or compressed beyond a certain limit, it will not regain its original state and will remain deformed. The limit beyond which permanent deformation occurs is called the elastic limit.

Bending of beams

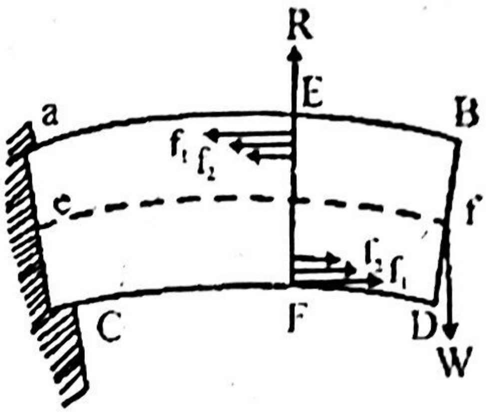
Beam

A beam is defined as a rod or bar of uniform cross section (circular or rectangular) whose length is very much greater than its thickness.

Bending couple

If a beam is fixed at one end and loaded at the other end, it bends. The load acting vertically downwards at its free end and the reaction at the support acting vertically upwards, constitute the bending couple. This couple tends to bend the beam clockwise. Since there is no rotation of the beam, the external bending couple must be balanced by another equal and opposite couple which comes into play inside the body due to the elastic nature of the body.

The moment of this elastic couple is called the internal bending moment. When the beam is in equilibrium, the external bending moment = the internal bending moment.



Plane of bending

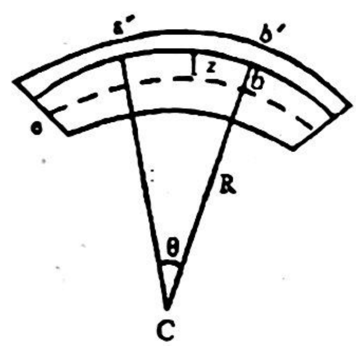
The plane of bending is the plane in which the bending takes place and the bending couple acts in this plane. In figure, the plane of paper is the plane of bending.

Neutral axis

When a beam is bent as in figure, filaments like ab in the upper part of the beam are elongated and filaments like cd in the lower part are compressed. Therefore, there must be a filament like ef in between, which is neither elongated nor compressed. Such a filament is known as the neutral filament and the axis of the beam lying on the neutral filament is the neutral axis. The change in length of any filament is proportional to the distance of the filament from the neutral axis.

Expression for the bending moment

Consider a portion of the beam to be bent into a circular arc as shown in figure. ef is the neutral axis. Let R be the radius of curvature of the neutral axis and θ the angle subtended by it at its centre of curvature C. Filaments above ef are elongated while filaments below ef are compressed. The filament ef remains unchanged in length.



Let a'b' be a filament at a distance z from the neutral axis. The length of this filament a'b' before bending is equal to that of the corresponding filament on the neural axis ab.

We have, original length = ab = Rθ

Its extended length = a'b' = (R + z)θ

Increase in the length = a'b' – ab

= (R + z)θ – Rθ = Zθ

Linear strain = Increase in length / original length

= Zθ/Rθ = Z/R

We know that Σmr^2 is the moment of inertia of the body, which is equal to = Mk^2 similarly, $\Sigma\delta A.z^2$ is called the geometrical moment of inertia of the cross section of the beam about an axis through its centre perpendicular to the plane of bending. It is written as equal to AK^2 i.e., $\Sigma\delta A.z^2 = AK^2$ ie., $\Sigma\delta A.z^2 = AK^2$.

(A = Area of cross section and k – radius of gyration). But the sum of moments of forces acting on all the filaments is the internal bending moment which comes into play due to the elasticity. Thus, bending moment of beam = qAK^2/R.

Notes :

i. For a rectangular beam of breadth b, and depth (thickness) d, $A = bd$ and $k^2 = d^2/12$

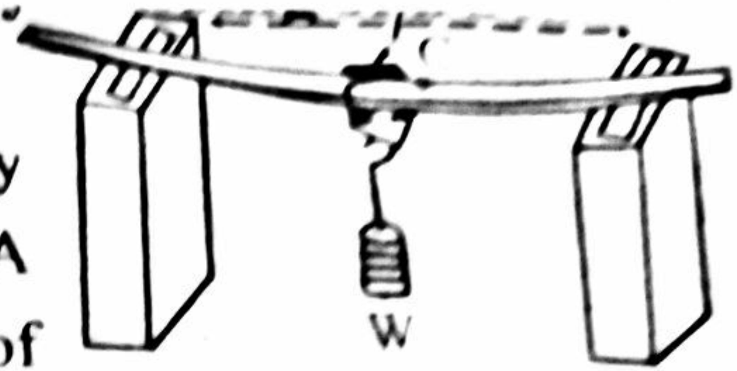
$AK^2 = bd^3/12$

ii. For a beam of circular cross section of radius r, $A = \pi r^2$ and $K^2 = r^2/4$.

$$AK^2 = \frac{\pi r^2}{4} \quad \text{Bending moment} = \frac{\pi q r^4}{4R}$$

Experimental determination of Young's Modulus pin and microscope method

1. Non – uniform bending



The given beam is symmetrically supported on two knife-edges figure. A weight hanger is suspended by means of a loop of thread from the point C exactly mid-way between the knife edges.

A pin is fixed vertically at C by some wax. A travelling microscope is focused on the tip of the pin such that the horizontal cross wire coincides with the tip of the pin. The reading in the vertical traverse scale of microscope is noted. Weights are added is equal steps of m kg and the corresponding readings are noted. Similarly, readings are noted while unloading. The results are tabulated as follows.

Load in kg	Reading of the microsope			y for Mkg
	Load Increasing	Load Decreasing	Mean	

The mean depression y is found for a load of M kg. The length of the beam (l) between the knife edged is measured. The breadth 'b' and thickness 'd' of the beam are measured with a vernier calipers and screw gauge respectively.

Then, $y = \frac{Wl^3}{48qAk^2}$

or $q = \frac{Wl^3}{48yAk^2}$

or $q = \frac{Mgl^3}{48(bd^3/12)xy}$ ($W = Mg$ and $Ak^2 = bd^3/12$)

$q = \frac{Mgl^3}{4bd^3y}$

Experimental method of young's modulus uniform bending

Pin and microscope method:

The given beam is supported symmetrically on two knife-edges A and B. Two equal weight hangers are suspended so that their distance from the knife edges are equal. A pin is placed vertically at the centre of the beam. The tip of the pin is viewed by a microscope. The load on each hanger is increased in equal steps of mkg and the corresponding microscope readings are noted while unloading. The results are tabulated as follows.

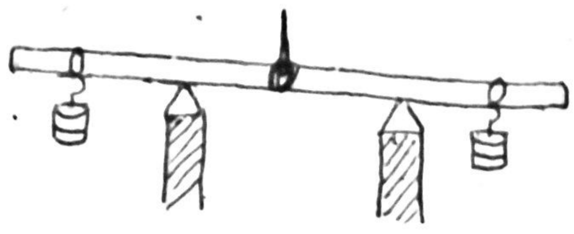
Load in kg	Reading of the microsope			y for Mkg
	Load Increasing	Load Decreasing	Mean	

The mean elevation (y) of the centre for Mkg is found. The length of the beam l between the knife edges and a the distance between the point of suspension of the load and the nearer knife-edge (AC = BD = a) are measured. The breadth b and the thickness d of the beam are also measured.

$$y = \frac{Wal^2}{8qAk^2} = \frac{Mgal^2}{8q(bd^3/12)} \qquad (W = Mg \text{ and } Ak^2 = bd^3/12)$$

$$q = \frac{3Mgal^2}{2bd^3y}$$

Using the above formula we can calculate the young's modulus of the material of the beam.



Expression for Young's Modulus non-uniform bending

Let AB represent a beam of length l, supported on two knife-edges at A and B and loaded with a weight W at the centre C. The reaction at each knife-edge is $W/2$ acting vertically upwards. The beam bends as shown in Fig. 1.18, the depression being maximum at the centre. The bending is non-uniform. Let $CD = y$.

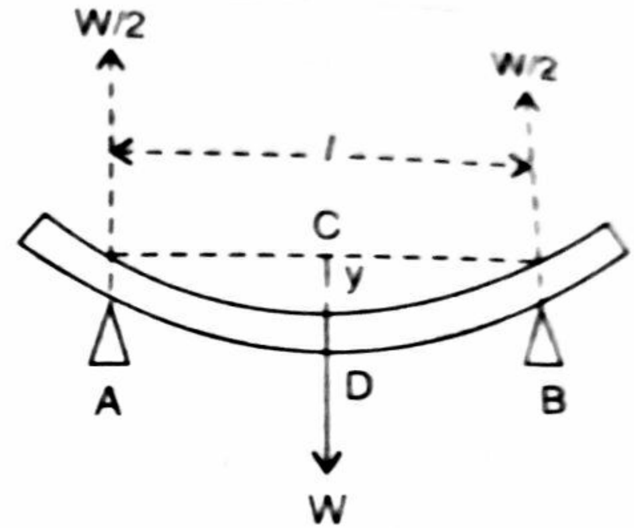


Fig 1.18

The portion DA of the beam may be considered as a cantilever of length $l/2$, fixed at D and bending upwards under a load $W/2$. Hence the elevation of A above D or,

$$\text{the depression of } D \text{ below } A = y = \frac{(W/2)(l/2)^3}{3EAk^2} = \frac{Wl^3}{48\,EAk^2}$$

Note : The inclination of the tangent at the points A and B is given by

$$\tan\theta = \frac{dy}{dx} = \frac{W\,l^2}{16\,EAk^2}$$

Since θ is small, $\tan\theta = \theta$.

$$\therefore \quad \theta = \frac{W\,l^2}{16\,EA\,k^2}$$

Experimental determination of Young's Modulus non-uniform bending

(1) Non-uniform Bending : The given beam is symmetrically supported on two knife-edges (Fig. 1.21). A weight-hanger is suspended by means of a loop of thread from the point C exactly midway between the knife-edges. A pin is fixed vertically at C by some wax. A travelling microscope is focussed on the tip of the pin such that the horizontal cross-wire coincides with the tip of the pin. The reading in the vertical traverse scale of microscope is noted. Weights are added in equal steps of m kg and the corresponding readings are noted. Similarly, readings are noted while unloading. The results are tabulated as follows :

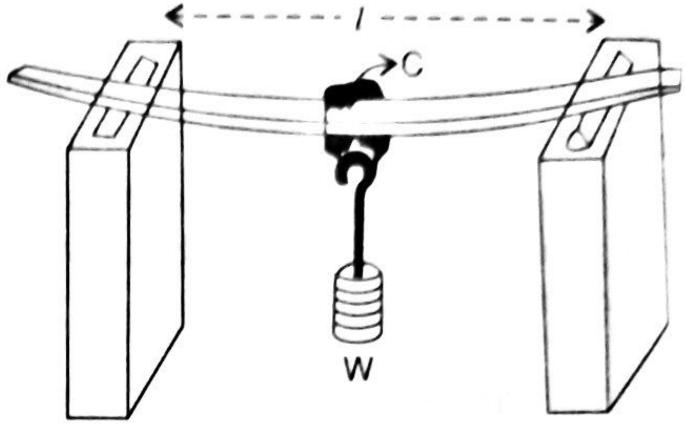


Fig. 1.21

Load in kg	*Readings of the microscope*			*y for M kg*
	Load increasing	*Load decreasing*	*Mean*	

The mean depression y is found for a load of M kg. The length of the beam (l) between the knife-edges is measured. The breadth b and the thickness d of the beam are measured with a vernier calipers and screw gauge respectively.

Then,
$$y = \frac{Wl^3}{48\,EA\,k^2} \text{ or } E = \frac{Wl^3}{48\,Ak^2 y}$$

or
$$E = \frac{Mg\,l^3}{48 \times (bd^3/12) \times y} \qquad (\because W = Mg \text{ and } Ak^2 = bd^3/12)$$

$\therefore$
$$E = \frac{Mgl^3}{4\,bd^3 y}$$

Expression for Young's Modulus Uniform bending

Consider a beam of negligible mass supported symmetrically on two knife-edges A and B in a horizontal level (Fig. 1.19). Let $AB = l$.

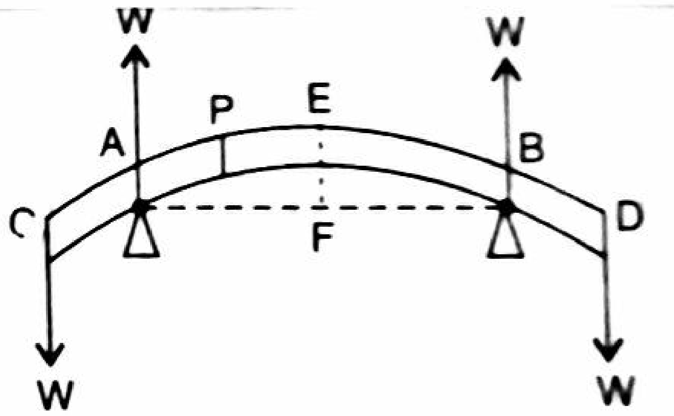


Fig. 1.19

Let equal weights W, W be added to the beam at its ends C and D. Let $AC = BD = a$. Then the beam is bent into an arc of a circle. The reactions on the knife-edges will then be W and W, acting vertically upwards. Consider the cross-section of the beam at any point P. The only forces acting on the part PC of the beam are the forces W at C and the reaction W at A.

The external bending moment with respect to P

$$= W \cdot CP - W \cdot AP = W(CP - AP) = W \cdot AC = Wa.$$

This must be balanced by the internal bending moment EAk^2/R.

Hence, $$Wa = EAk^2/R \qquad \ldots(1)$$

Since for a given load W, E, a and Ak^2 are constant, R is a constant. The bending is then said to be uniform. If y is the elevation of the mid-point of AB above its normal position (Fig. 1.20),

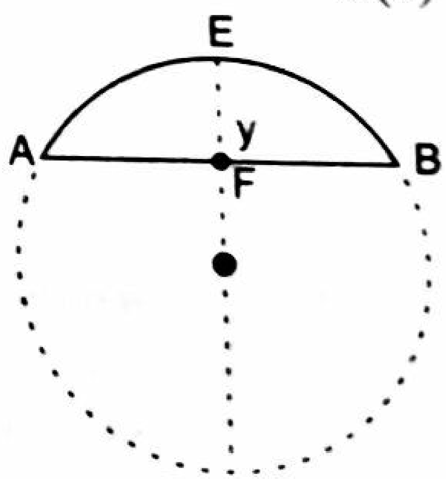


Fig. 1.20

$$EF(2R - EF) = AF^2$$

$$y(2R - y) = (l/2)^2$$

$$y \cdot 2R = l^2/4 \qquad (\because y^2 \text{ is negligible})$$

$$y = l^2/8R$$

Uniform bending : The given beam is supported symmetrically on two knife-edges *A* and *B* (Fig. 1.22). Two equal weight-hangers are suspended, so that their distances from the kinfe-edges are equal. The elevations of the centre of the beam may be measured accurately by using a single optic level (*L*). The front leg of the single optic lever rests on the centre of the loaded beam and the hind legs are supported on a separate stand. A vertical scale (S) and telescope (T) are arranged in front of the mirror. The telescope is focussed on the mirror and adjusted so that the reflected image of the scale in the mirror is seen through the telescope. The load on each hanger is increased in equal steps of m kg and the corresponding readings on the scale are noted. Similarly, readings are noted while unloading. The results are tabulated as follows :

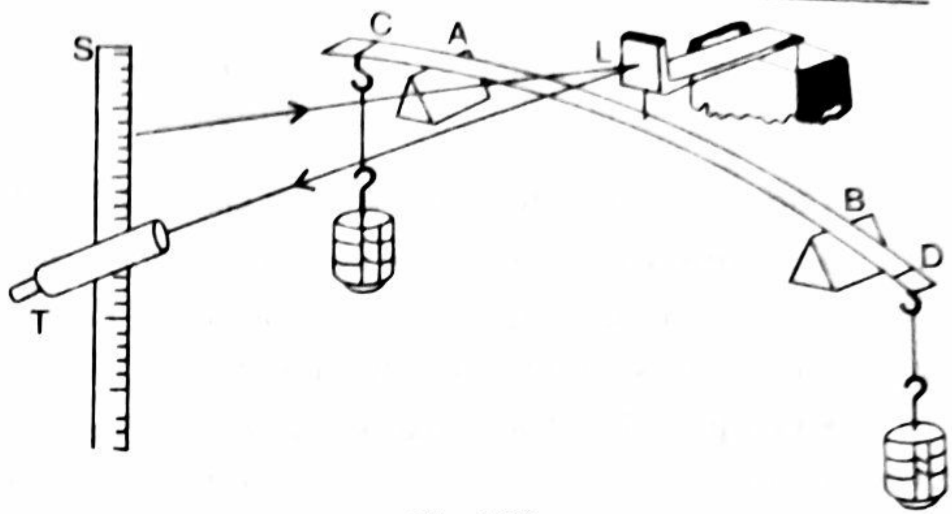


Fig. 1.22

Load in kg	*Readings of the scale as seen in the telescope*			*Shift in reading for M kg*
	Load increasing	*Load decreasing*	*Mean*	

The shift in scale reading for M kg is found from the table. Let it be S. If

D = The distance between the scale and the mirror,

x = the distance between the front leg and the plane containing the two hind legs of the optic lever,

then $$y = Sx/2D.$$

The length of the beam l between the knife-edges, and a, the distance between the point of suspension of the load and the nearer knife-edge ($AC = BD = a$) are measured. The breadth b and the thickness d of the beam are also measured.

Then, $$y = \frac{Wal^2}{8\,E\,A\,k^2} \quad \text{or} \quad \frac{Sx}{2D} = \frac{Mgal^2}{8\,E\,(b\,d^3/12)}$$

[Since $W = Mg$ and $Ak^2 = b\,d^3/12$]

$$\therefore \quad E = \frac{3\,Mgal^2 D}{Sxbd^3}$$

Torsion of a body

When a body is fixed at one end and twisted about its axis by means of a couple at the other end, the body is said to be under torsion. Torsion involves shearing strain and so the modulus involved is the rigidity modulus.

Torsion of a wire

Expression for couple per unit Twist

Consider a cylindrical wire of length L and radius fixed at its upper end and twisted through an angle θ by applying a couple at the lower end. Consider the cylinder to consist of an infinite number of hollow co-axial cylinders. Consider one such cylinder of radius x and thickness dx figure.

A line such as AB initially parallel to the axis OO' of the cylinder is displaced to the position AB' through an angle ϕ due to the twisting couple figure. The result of twisting the cylinder is a shear strain. The angle $\phi$of shear = $\angle BAB' = \phi$

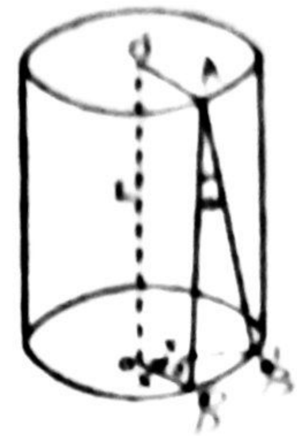
Now $BB' = x.\theta = L\phi$ or $\phi = x.\theta/L$

$$\text{We have, rigidity} = n = \frac{\text{Shearing stress}}{\text{Angle of shear } (\phi)}$$

Shearing stress $= n.\ \phi$ or $\phi = nx.\theta/L$

$$\text{But, Shearing stress} = \frac{\text{Shearing force}}{\text{Area of which the force acts}}$$

Shearing force = Shearing stress x Area on which the force acts.



The area over which the shearing force acts = $2\pi x\, dx$

Hence, the shearing force $= F = \dfrac{nx\theta}{L} \times 2\pi x\, dx$

The moment of this force about the axis OO' of the cylinder $= \dfrac{nx\theta}{L} \times 2\pi \times dx \cdot x$

$$= \frac{2\pi n\theta}{L} x^3\, dx$$

Twisting couple on the whole cylinder $= C = \displaystyle\int_0^a \frac{2\pi n\theta}{L} x^3 dx$

$$= C = \frac{\pi n a^4 \theta}{2L}$$

The couple per unit twist (i.e., the couple when $\theta = 1$ radian)

$$= C = \frac{\pi n a^4 \theta}{2L}$$

Note : 1

When an external couple is applied on the cylinder to twist it at once an internal couple, due to elastic force comes into play. In the equilibrium position, these two couples will be equal and opposite.

Note : 2

If the material is in the form of a hollow cylinder of internal 'a' radius and external radius 'b', then

$$\text{Couple per unit twist} = C = \int_0^a \frac{2\pi n\theta}{L} x^3 dx$$

$$= \frac{\pi n\theta}{2L}(b^4 - a^4)$$

Couple per unit twist = $c = \pi n(b^4 - a^4)/2L$

Work done in twisting a wire

Consider a cylindrical wire of length L and radius 'a' fixed at its upper end and twisted through an angle θ by applying a couple at the lower end.

If c is the couple per unit angular twist of the wire, then the couple required to produce a twist 0 in the wire is $C = c\theta$.

The work done in twisting the wire through a small angle $d\theta$ is

$$Cd\theta = c\theta\, d\theta$$

The total work done in twisting the wire through an angle θ.

$$= W = \int_0^\theta c\theta \, d\theta$$

$$= \frac{1}{2} c\theta^2$$

The work done in twisting the wire is stored up in the wire a potential energy.

TORSIONAL OSCILLATIONS OF A BODY

Suppose a wire is clamped vertically at one end and the other end carries a body (i.e., a disc, bar or a cylinder) of moment of inertia I about the wire as the axis. Let the length, radius and rigidity modulus of the wire be respectively l, a and G. When the body is given a slight rotation by applying a torque, say by the hand, the wire is twisted. If the body is released, the body oscillates in the horizontal plane about the wire as axis. These oscillations are called *Torsional oscillations* and the arrangement is known as a *Torsion pendulum*.

Let us consider the energy of the vibrating system when the angle of twist is θ. Let ω be the angular velocity of the body.

The potential energy of the wire due to the twist $= \frac{1}{2} c . \theta^2$.

The kinetic energy of the body due to its rotation $= \frac{1}{2} I \omega^2 = \frac{1}{2} I \left(\frac{d\theta}{dt}\right)^2$

The total energy of the system $= \frac{1}{2} I \left(\frac{d\theta}{dt}\right)^2 + \frac{1}{2} c\theta^2 = \text{constant}$

Differentiating this with respect to t,

$$\frac{1}{2} I . 2 \frac{d\theta}{dt} \cdot \frac{d^2\theta}{dt^2} + \frac{1}{2} c \, 2\theta \frac{d\theta}{dt} = 0.$$

or
$$I \frac{d^2\theta}{dt^2} + c\theta = 0 \text{ or } \frac{d^2\theta}{dt^2} + \frac{c}{I} \theta = 0$$

The body has simple harmonic motion and its period is given by

$$T = 2\pi \sqrt{\frac{I}{c}}$$

Rigidity modulus by Torsion pendulum (Dynamic torsion method) :

The torsion pendulum consists of a wire with one end fixed in a split chuck and the other end to the centre of a circular disc as in Fig. 1.10.

Two equal symmetrical masses (each equal to m) are placed along a diameter of the disc at equal distances d_1 on either side of the centre of the disc. The disc is rotated through an angle and is then released. The system executes torsional oscillations about the axis of the wire. The period of oscillations T_1 is determined.

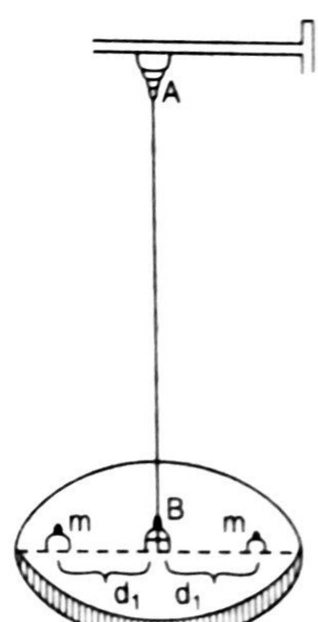


Fig. 1.10

Then
$$T_1 = 2\pi \sqrt{\frac{I_1}{c}}$$

or
$$T_1^2 = \frac{4\pi^2}{c} I_1.$$

Here, I_1 = Moment of inertia of the whole system about the axis of the wire and

c = torque per unit twist.

Let I_0 = M.I. of the disc alone about the axis of the wire.

i = M.I. of each mass about a parallel axis passing through its centre of gravity.

Then by the parallel axes theorem,

$$I_1 = I_0 + 2i + 2md_1^2$$

$$\therefore \quad T_1^2 = \frac{4\pi^2}{c} [I_0 + 2i + 2m \,.\, d_1^2]. \qquad ...(1)$$

The two masses are now kept at equal distances d_2 from the centre of the disc and the corresponding period T_2 is determined. Then,

$$T_2^2 = \frac{4\pi^2}{c} [I_0 + 2i + 2md_2^2] \qquad ...(2)$$

$$\therefore \quad T_2^2 - T_1^2 = \frac{4\pi^2}{c} \cdot 2m \cdot (d_2^2 - d_1^2) \qquad ...(3)$$

But $$c = \pi Ga^4/2L$$

Hence $$T_2^2 - T_1^2 = \frac{4\pi^2\, 2m\,(d_2^2 - d_1^2)\, 2L}{\pi Ga^4}$$

or $$G = \frac{16\,\pi\, Lm\,(d_2^2 - d_1^2)}{a^4\,(T_2^2 - T_1^2)}$$

Using this relation, G is determined.

M.I. of the disc by torsional oscillations. The two equal masses are removed and the period T_0 is found when the disc alone is vibrating. Then,

$$T_0^2 = \frac{4\,\pi^2}{c} I_0 \text{ or } I_0 = \frac{cT_0^2}{4\,\pi^2} \qquad ...(4)$$

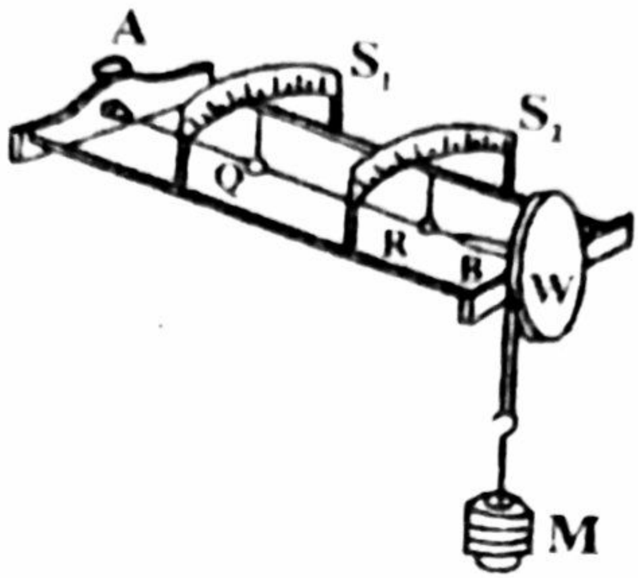
From (3), $$c = \frac{4\,\pi^2\, 2m\,(d_2^2 - d_1^2)}{T_2^2 - T_1^2}$$

Hence $$I_0 = \frac{4\,\pi^2\, 2m\,(d_2^2 - d_1^2)}{T_2^2 - T_1^2} \cdot \frac{T_0^2}{4\,\pi^2} = \frac{2\,m\,(d_2^2 - d_1^2)\,T_0^2}{T_2^2 - T_1^2}$$

From this relation, the moment of inertia of the disc about the axis of the wire is calculated.

Determination of rigidity modulus -Static torsion method ; Searle's apparatus :

The experimental rod is rigidly fixed at one end A and fitted into the axle of a wheel W at the other end B figure.



The wheel is provided with a grooved edge over which passes a tape. The tape carries a weight hanger at its free end. The rod can be twisted by adding weights to the hanger. The angle of twist can be measured by means of two pointers fixed at Q and R. Which move over circular scales S_1 and S_2. The scales are marked in degrees with centre zero.

When no weights on the, hanger, the initial readings the pointers on the scales are adjusted to be zero. Loads are added in steps of m kg (conveniently 0.2 kg). The readings on the two scales are noted for every load, both while loading and unloading. The experiment is repeated after reversing the twisting couple by winding

Load	Reading on S_1					Reading on S_2					$\theta = \theta_2 - \theta_1$	θ for Mkg
	Torque Clockwise		Torque Anticlockwise			Torque Clockwise		Torque Anticlockwise				
	Load increasing	Load decreasing	Load increasing	Load decreasing	Mean θ_1	Load increasing	Load decreasing	Load increasing	Load decreasing	Mean θ_2		

the tape over the wheel in the opposite way. The observations are tabulated .

The reading in the last column give the twist for a load of M Kg for the length QR (=L) of the rod.

The radius a of the rod and the radius R of the wheel are measured.

If a load of M kg is suspended from the free end of the tape, the twisting couple = Mg R.

The angle of twist $= \theta$ degrees $= \theta.\pi/180$ radians.

The restoring couple $$= \frac{\pi n a^4}{2L}\frac{\theta\pi}{180}$$

The equilibrium MgR $$= \frac{\pi n a^4}{2L}\frac{\theta\pi}{180} \text{ or } n = \frac{360\ M\ g\ RL}{\pi^2 a^4 \theta}$$

Since occurs in the fourth power in the relation used it should be measured very accurately.

Notes

1. We eliminate the error due to the eccentricity of the wheel by applying the couple in both clockwise and anticlockwise directions.
2. We eliminate errors due to any slipping at the clamped end by observing reading at two points on the rod.

Surface Tension

Definition

It may be defined as the force per unit length of it a line drawn in the liquid surface acting perpendicular to it at every point and tending to pull the surface a part along the line.

Unit of surface tension

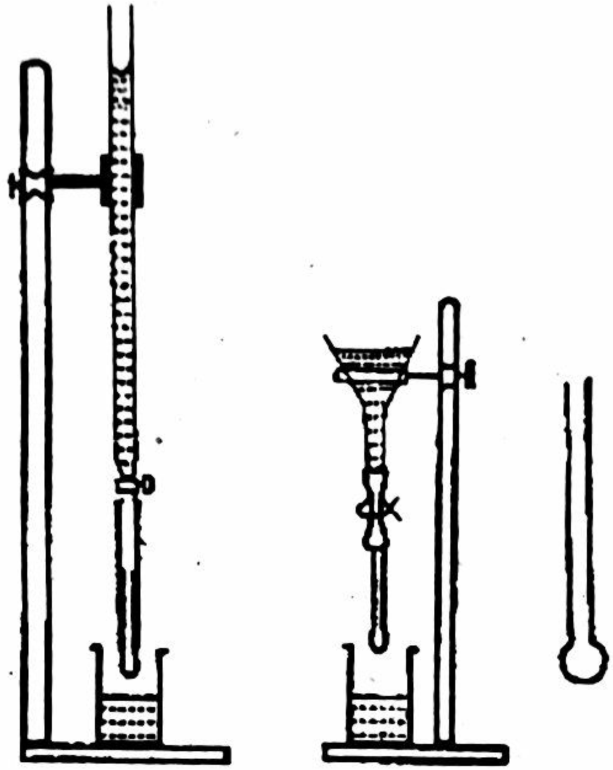
Surface tension being force per unit length, its SI unit is Newton per metre (Nm^{-1}).

Dimensions of Surface Tension

Since it is the ratio of a force to a length, its dimension are $MLT^{-2}/ L = MT^{-2}$

Drop Weight method of determining the surface tension of a liquid

Experiment



A short glass tube is connected to the lower end of a burette (or funnel) clamped vertically by means of a rubber tube (figure). The funnel is filled with the liquid whose surface tensions is to be determined. A beaker is arranged under the glass tube to collect the liquid dropping from the funnel. The stopcock is adjusted so that the liquid drops are formed slowly. In a previously weighted beaker a known number of drops are collected.

The beaker is again weighted. The difference between this weight and the weight of the empty beaker gives the weight of 50 drops of the liquid. From this the mass m of each drop is calculated. The inner radius r of the tube is determined using a vernier calipers. The surface tension of the liquid at the room temperature is calculated using the formula. $\sigma = mg/38r$

Theory

Here, we consider the vertical forces that keep a small drop of liquid in equilibrium, just before it gets detached from the end of a vertical glass tube of circular aperture. At the instant the drop gets detached, it assumes a cylindrical shape at the orifice of the tube. Let σ = ST of the liquid and r = radius of the orifice.

Excess pressure (p) inside the drop over the outside atmospheric pressure = σ/r

The area of the cross section is πr^2. Therefore, downward force on the drop due to this excess of pressure = $\pi r^2 - \sigma/r$

The weight mg of the drop also acts vertically downwards.

Total downward force on the drop = $\pi r^2 \sigma/r + mg$

This downward force is balanced by the upwards pull due to surface tension zero acting along a circle of radius r. Therefore

$$2\pi r\sigma = \pi r^2\sigma/r + mg \text{ or } 2\pi r\sigma = \pi r\sigma + mg$$

$$\sigma = mg/\pi r$$

But the equilibrium of the drop at the instant of its detachments is dynamic and not static. Lord Rayleighty taking dynamical aspect into account showed that $\sigma = mg/3.8r$

Interfacial tension

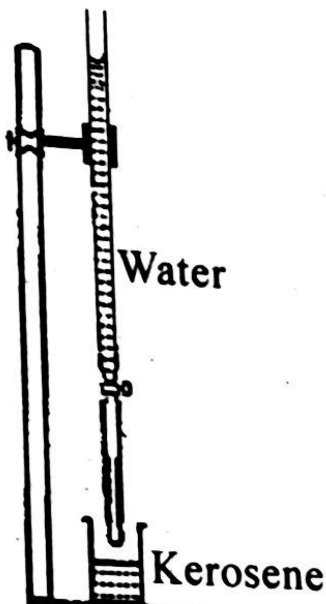
At the surface of separation between two immiscible liquid there is a tension similar to surface tension. It is called the interfacial tension.

Definition

When one liquid rests on another without mixing with it the interface between the two liquids possesses energy just like the surface of a liquid. The interfacial tension is the value of the force acting per metre normal to a line drawn on the interface.

Experiment to determine the interfacial tension between water and kerosene

Sufficient amount of the lighten liquid (kerosene) is taken in a beaker. The weight w_1 of the baker with kerosene is determined. The heavier liquid (water) is taken in the burette (figure). The glass tube is fixed vertically with its end under the surface of kerosene. The flow of water is regulated so the drops of water detach themselves into kerosene one by one. After collecting 50 drops, the beaker is again weighted. Let this weight be w_2. Then $w_2 - w_1$ gives the mass of 50 drops. From this the average mass m of each drop is calculated.



The interfacial tension σ between water and kerosene is calculated using the formula.

$$\sigma = \frac{mg}{3.8r}\left[1 - \frac{\rho_2}{\rho_1}\right]$$

Theory

Let ρ_1 and ρ_2 be the densities of water and kerosene respectively. Let m be the mass of water drop in air. Volume of water drop = m/ρ_1.

Volume of kerosene displaced by the water drop = $\frac{mg}{\rho_1}$

Mass of kerosene displaced by the water drop = $\frac{m\rho_2}{\rho_1} \cdot g$

Apparent weight of the water drop in kerosene = $mg - \frac{m\rho_2 g}{\rho_1}$

Let σ be the surface tension at the interface between the two liquids.

$$\text{Then } 2\pi r = \frac{\pi r 2\sigma}{r} + mg - \frac{m\rho_2 g}{\rho_1}$$

$$\sigma = \frac{mg}{nr\,(1 - (\rho_2/\rho_1))}$$

Again the more accurate equation a will be

$$\sigma = \frac{mg}{3.8r\,(1 - (\rho_2/\rho_1))}$$